

DEEP INELASTIC NEUTRINO INTERACTIONS

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Next-to-leading order QCD corrections, order 1 GeV mass corrections and the role of a strangeness asymmetry and isospin violation in the x dependence of parton distributions are evaluated in the context of the neutrino-nucleon cross section. Their contributions to evaluations of the weak mixing angle using the Paschos-Wolfenstein relation are discussed.

1. Introduction

Neutrino scattering with nucleons is a well-studied phenomenon in a variety of energy regimes.¹ Because of the results on neutrino oscillations determined by the Super-Kamiokande experiment and others from their studies of atmospheric neutrino fluxes,² tau neutrino scattering as well as muon neutrino and electron neutrino scattering is considered. In another arena of neutrino physics, the NuTeV experimental collaboration's precision work on neutrino scattering and their determination of $\sin^2 \theta_W$ has led to a reexamination of the neutrino cross section. Their result,³ that

$$\sin^2 \theta_W = 0.2277 \pm 0.0013 \pm 0.0009 , \quad (1)$$

is in contrast to the world average without NuTeV:⁴ 0.2227 ± 0.0004 , a difference of ~ 0.005 . In both of these cases, one is led to reconsider the role of next-to-leading order (NLO) perturbative QCD and the role of the mass corrections of order $O(\text{GeV})$. Here, we focus on the case of the NuTeV result for $\sin^2 \theta_W$, discussing the impact of the perturbative QCD corrections^{5,6,7,8} and mass corrections,⁵ and considering the role of a strangeness asymmetry and isospin violation in the x dependence of the parton distribution functions in resolving the discrepancy.^{7,8,9}

2. NLO QCD and Mass Corrections

The starting point for the neutrino-nucleon cross section, including NLO QCD and mass corrections, is the differential cross section in terms of $x = Q^2/(2P \cdot q)$, $y = (E_\nu - E_\ell)/E_\nu$ and $q = p_\nu - p_\ell$ with $q^2 = -Q^2$. For M representing the nucleon

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mass and m the lepton mass, the charged current differential cross section is

$$\begin{aligned} \frac{d^2\sigma_{CC}^\nu(\nu N)}{dx dy} = & \frac{G_F^2 M E_\nu}{\pi(1 + Q^2/M_W^2)^2} \left[\left(xy^2 + \frac{m^2 y}{2M E_\nu} \right) F_1^{TMC} \right. \\ & + \left(1 - \frac{m^2}{4E^2} - y - \frac{Mxy}{2E_\nu} \right) F_2^{TMC} + \left(x - \frac{xy^2}{2} - \frac{m^2 y}{4M E_\nu} \right) F_3^{TMC} \\ & \left. + \left(\frac{m^2 y}{2M E_\nu} + \frac{m^4}{4M^2 E_\nu^2 x} \right) F_4^{TMC} - \frac{m^2}{M E_\nu} F_5^{TMC} \right]. \end{aligned} \quad (2)$$

Lepton and target masses also enter through the limits on x and y . Charm mass corrections from $W^+ + s \rightarrow c$ are incorporated in the structure functions.^{5,10,11} QCD and target mass corrections are included in the structure functions $F_i^{TMC} = F_i^{TMC}(\xi, Q^2)$, where ξ is the Nachtmann variable defined by

$$\frac{1}{\xi} = \frac{1}{2x} + \sqrt{\frac{1}{4x^2} + \frac{M^2}{Q^2}}. \quad (3)$$

The target mass corrected structure functions have target mass dependence through ξ , and they have corrections related to the mismatch between partonic and hadronic tensor expansions of the interaction vertex squared.¹² The final M^2/Q^2 corrections come from parton intrinsic transverse momentum effects which are limited by M .¹³ Combined, these give the same results as the operator product expansion [OPE] discussed by Georgi and Politzer.¹⁴ The results, for example, for F_2^{TMC} is

$$\begin{aligned} F_2^{TMC} = & 2 \frac{x^2}{\rho^3} \frac{\mathcal{F}_2(\xi, Q^2)}{\xi} + 12 \frac{M^2}{Q^2} \frac{x^3}{\rho^4} \int_\xi^1 d\xi' \frac{\mathcal{F}_2(\xi', Q^2)}{\xi'} \\ & + 24 \frac{M^4}{Q^4} \frac{x^4}{\rho^5} \int_\xi^1 d\xi' \int_{\xi'}^1 \frac{\mathcal{F}_2(\xi'', Q^2)}{\xi''}, \end{aligned} \quad (4)$$

in terms of $\rho^2 = 1 + 4M^2 x^2/Q^2$ and $\mathcal{F}_2 = q(\xi, Q^2) + \bar{q}(\xi, Q^2)$.

3. Approach to $\sin^2 \theta_W$

The NuTeV experimental approach to extracting $\sin^2 \theta_W$ uses both $\nu_\mu N$ and $\bar{\nu}_\mu N$ scattering. Their analysis uses correlated the correlated R_ν and $R_{\bar{\nu}}$ measurements where

$$R_{\nu, \bar{\nu}} \equiv \frac{\sigma_{NC}^{\nu, \bar{\nu}}}{\sigma_{CC}^{\nu, \bar{\nu}}} \quad (5)$$

in terms of the charged current (CC) and neutral current (NC) cross sections. A particularly useful theoretical quantity is

$$R^- \equiv \frac{\sigma_{NC}^\nu - \sigma_{NC}^{\bar{\nu}}}{\sigma_{CC}^\nu - \sigma_{CC}^{\bar{\nu}}} \quad (6)$$

where many uncertainties cancel. In addition, R^- is fairly independent of energy: the beam of neutrinos (and antineutrinos) is distributed in energy. Theoretically, with several approximations:

- assume isoscalar nucleons with $u(x) = d(x) \equiv q(x)$ and the usual isospin relations between up and down quark distributions in the proton and neutron,
- neglect the charm mass so $s \rightarrow c$ not suppressed (Cabbibo angles not relevant)
- neglect target masses and work in leading order QCD
- take $s(x) = \bar{s}(x)$, etc., for the sea quark distributions,

one gets the Paschos-Wolfenstein relation¹⁵

$$R^- = \frac{(1 - 2 \sin^2 \theta_W)}{2} . \quad (7)$$

With these approximations, Eq. (7) is independent of the limits of integration on x and y which could be used to mock up experimental cuts.

In terms of an actual measurement of R^- , pure neutral current and charge current event samples are not possible. In addition, there are the effects of cuts. The electron neutrino background is subtracted. While the NuTeV measurement is not a direct measurement of R^- , it is a useful theoretical effort to examine NLO QCD corrections to R^- to assess their impact on the extraction of $\sin^2 \theta_W$.

4. Corrections to R^-

4.1. NLO QCD

We first discuss the NLO perturbative QCD corrections to R^- including target mass, lepton mass and charm mass corrections. As noted above, it is only with some approximations that R^- is simply related to $\sin^2 \theta_W$ as in Eq. (7), so our approach is to look at the full NLO QCD corrections to $1/2 - R^-$ and to compare with the LO evaluation. An approximate moment analysis of the NLO corrections appears in Refs. 6-8. The full evaluation of the NLO corrections leads to the results in Table 1. We show results with an input $\sin^2 \theta_W$ of 0.2227, using the Gluck, Reya and Vogt parton distribution functions¹⁶ (PDFs), and the CTEQ6 PDFs,¹⁷ including the 40 sets with individual variations in the 20 parameters in the sets to estimate the error. To account for the fluxes of neutrinos and antineutrinos,

$$\sigma_{\text{NC,CC}}^{\nu,\bar{\nu}} = \frac{\int dE_{\nu,\bar{\nu}} d\sigma_{\text{NC,CC}}^{\nu,\bar{\nu}} \Phi(E_{\nu\bar{\nu}})}{\int dE_{\nu,\bar{\nu}} \Phi(E_{\nu\bar{\nu}})} \quad (8)$$

for incident neutrino/antineutrino flux Φ . Details of the flux used appear in Ref. 5. To mimic some experimental cuts, we take $20 \text{ GeV} < yE_{\nu,\bar{\nu}} < 180 \text{ GeV}$ in Eq. (8). Table 1 shows that the NLO corrections cannot account for the discrepancy between the NuTeV evaluation of $\sin^2 \theta_W$ and the world average.

Table 1. NLO perturbative QCD corrections to R^- .⁵

Input PDF	Input $\sin^2 \theta_W$	$\frac{1}{2} - R^-$
GRV LO	0.2227	0.2192
GRV NLO	0.2227	0.2192
CTEQ6 NLO	0.2227	0.2196
		± 0.0005

4.2. Strange sea asymmetry

So far, we have assumed isospin symmetry and $q(x) = \bar{q}(x)$, however, one can relax the second condition for strange sea. It is reasonable that $s(x) \neq \bar{s}(x)$ since the s can arrange itself in mesons and baryons, while the \bar{s} can only go into meson fluctuations.¹⁸ The condition that the net strangeness of the nucleon vanishes must still be satisfied:

$$\int_0^1 dx (s(x) - \bar{s}(x)) = 0, \quad (9)$$

However, one can have

$$[S^-] \equiv \int dx x [s(x) - \bar{s}(x)] \neq 0. \quad (10)$$

In terms of $[S^-]$, the implication for R^- is that

$$R^- \simeq \frac{1}{2} - \sin^2 \theta_W - \left(\frac{1}{2} - \frac{7}{6} \sin^2 \theta_W \right) \frac{[S^-]}{[Q^-]} \quad (11)$$

with isoscalar up or down quark distributions $q(x)$ contributing via $[Q^-] = \int x [q(x) - \bar{q}(x)] dx$. A positive value for $[S^-]$ works in the direction to moderate the disagreement between NuTeV and the world average values of $\sin^2 \theta_W$.

Olness et al.¹⁹ have performed a global analysis of dimuon production $[W^+ s \rightarrow c$ and $W^- \bar{s} \rightarrow \bar{c}$ with semileptonic charm decay] together with the other PDFs. Their results favor²⁰ $[s(x) - \bar{s}(x)] < 0$ at low- x , and $[s(x) - \bar{s}(x)] > 0$ at large- x , so $[S^-] > 0$. The allowed values for $[S^-]$ range between¹⁹

$$-0.001 < [S^-] < 0.004. \quad (12)$$

A recent paper by Catani, de Florian, Rodrigo and Vogelsang²¹ shows that an asymmetry of $[S^-] \sim -5 \times 10^{-4}$ is generated by NNLO perturbative evolution, even with a LO value of $[S^-] = 0$. The implication is that the strangeness asymmetry must be of nonperturbative origin if it is as large as in Eq. (12). We note that the NuTeV experimental collaboration reports a small negative value for $[S^-]$.²²

Using our full NLO QCD evaluation of the cross sections, we evaluate R^- using several of the PDFs with a strangeness asymmetry from Ref. 19 (labeled A, B, C, B⁺ and B⁻) and compare them to the central value of R^- using the CTEQ6 PDFs. Defining

$$\delta R^- \equiv R^-_{\{A,B,C,B^+,B^-\}} - R^-_{\text{CTEQ6}},$$

Table 2. Shifts in R^- calculated⁹ with the PDF sets of Olness et al.¹⁹ compared to the CTEQ6M set with $[S^-] = 0$.

fit	$[S^-] \times 100$	δR^-
B ⁺	0.540	-0.0065
A	0.312	-0.0037
B	0.160	-0.0019
C	0.103	-0.0012
B ⁻	-0.177	0.0023

our results appear in Table 2. A positive value of $[S^-]$ gives a negative value for δR^- . Our conclusion is that a nonperturbative input of $[S^-] > 0$ consistent with a global PDF analysis could explain the discrepancy in the $\sin^2 \theta_W$ measurements.^{8,9,23}

4.3. Isospin violation

Isospin symmetry violation,²⁴ in which the valence quark distributions do not obey the symmetry of $u_V^n(x) \neq d_V^p(x)$, etc., could also account for the discrepancy in $\sin^2 \theta_W$. One finds approximately that

$$\delta R_I^- \simeq -\left(\frac{1}{2} - \frac{7}{6} \sin^2 \theta_W\right) \frac{[D_N^- - U_N^-]}{[Q^-]}, \quad [D_N^-] = \frac{1}{2}([D_p^-] + [D_n^-]) \quad (13)$$

The MRST²³ PDFs yield

$$-0.007 < \delta R_I^- < 0.007 \quad (14)$$

$$\delta R_I^- = -0.0022 \quad \text{for the best fit ,}$$

with the best fit again working in the direction to moderate the discrepancy in $\sin^2 \theta_W$ values.

5. Summary

NLO QCD can't account for the discrepancy between NuTeV and other experimental evaluations of $\sin^2 \theta_W$, however, a strange-antistrange asymmetry that preserves net strangeness zero in the nucleon, consistent with a global PDF analysis of the data, moderates the discrepancy. Isospin violation may also come into play at the same level. Our results suggest that the Weinberg angle measurements may be accommodated within the standard model as long as parton distribution functions show strangeness and/or isospin asymmetries of nonperturbative origin at the level described in Table 2 and Eq. (14).

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